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B70 02053

SUBJECT: A Study of the Phasing Capability of Apollo CSI/CDH Coelliptic Rendezvous Targeting Routines for AAP - Case 610

DATE: February 19, 1970

FROM: C. O. Guffee
R. C. Purkey

ABSTRACT

This memorandum summarizes a study of the CSI/CDH rendezvous profile as it could be applied to AAP. The study has been performed using both conic equations of motion and a FORTRAN version of the Apollo on-board CSI/CDH and TPI targeting routines. The purpose of the study has been to define the phasing capability of the CSI/CDH rendezvous profile.

The principal conclusions are:

1. The time from CSI to CDH can be approximated by 2755 seconds multiplied by the number (N) of half periods between CSI and CDH. Since it is desirable to perform TPI at a particular time in order to preserve terminal phase lighting, and since it is most efficient to perform CSI at an apsidal crossing, a desired time from CSI to TPI can be determined. Then the time spent in coelliptic orbit is determined by the desired CSI to TPI time minus the time from CSI to CDH (N x 2755).

2. In order to allow 1800 seconds in coelliptic orbit, the CSI point must be at least 96 nm behind the target vehicle to allow for one half period from CSI to CDH, and at least 141 nm behind in order to allow for two half periods from CSI to CDH.

3. For each additional second desired in coelliptic orbit, the distance behind at CSI must be increased by 0.01695 nm for both the one and two half period profiles.

4. Conclusions 2 and 3 as related to the profile with one half period from CSI to CDH are for the case of CSI occurring at the coelliptic altitude. For each nautical mile that the CSI altitude is lowered, the distance behind at CSI must increase by 2.33 nm.

5. Conclusions 2 and 3, as related to the profile with two half periods from CSI to CDH, are for the case where the perigee of the CSI to CDH orbit is at the coelliptic altitude. For each 5 nm of difference in distance behind at CSI, the perigee altitude must be changed by 1 nm. Increasing the distance behind at CSI lowers the required perigee altitude.

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MEMORANDUM FOR FILE

I. Introduction

This memorandum summarizes a study of the CSI/CDH and TPI coelliptic rendezvous profile as it could be applied to AAP rendezvous. The primary purpose has been to define the phasing capability and effectiveness of the CSI/CDH type of rendezvous profile. In this way future comparisons with other rendezvous targeting routines, suitable for AAP, will be possible.* The study has been performed in conjunction with tests of a UNIVAC 1108 version of the Apollo on-board targeting routines.**

The rendezvous trajectories for AAP CSM missions will require several orbits and include phasing and/or height maneuvers followed by a coelliptic rendezvous profile. The inclusion of phasing and height maneuvers is to allow for longer launch windows, and the CSI/CDH maneuvers then act in a vernier capacity to insure arrival at a nominal TPI point, and thus provide a consistent terminal phase for the rendezvous. This study was made to determine the ability of the CSI/CDH rendezvous profile to effect a rendezvous when given phasing dispersions in relative downrange position with respect to the target vehicle at CSI time and dispersions in time of arrival at the CSI point. Thus the study is designed to determine the rendezvous phasing capability of the CSI/CDH rendezvous profile rather than a sensitivity analysis of small position and velocity errors and Δv execution errors.

Section II contains a description of the CSI/CDH and TPI targeting routines, and identifies the input and output data. Section III describes the rendezvous problem which has been investigated in this memorandum, and Section IV contains

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*The authors are presently performing studies with other coelliptic targeting routines and will report these results in future memoranda.

**The UNIVAC 1108 version of the CSI/CDH and TPI targeting routines are described in Apollo GSOP[1] and have been written jointly by the authors and G. J. Miel (2011).

a general description of two possible rendezvous profiles which can be targeted with the CSI/CDH routines. Sections V and VI discuss analytical and simulation results of these two profiles and Section VII discusses the conclusions.

II. CSI/CDH and TPI Targeting

This section is intended to give a brief description of the maneuvers associated with a CSI/CDH-targeted coelliptic rendezvous. References (1-4) provide additional details and, in particular, Reference (3) describes the philosophy of the coelliptic rendezvous profile.

The Apollo on-board targeted coelliptic rendezvous consists of three maneuvers:

1. Coelliptic Sequence Initiation (CSI) - a phasing and/or CDH height adjustment maneuver.

2. Constant Differential Altitude (CDH) - a maneuver to place the active vehicle into a coelliptic orbit with respect to the target orbit.

3. Terminal Phase Initiation (TPI) - a maneuver that terminates the coelliptic orbit and establishes an intercept path.

The inputs to the CSI targeting routines are the time of the CSI maneuver, the number of half periods between CSI and CDH, the desired line-of-sight angle from the active to the passive vehicle at TPI (measured from the horizontal plane of the active vehicle), and the time of the TPI maneuver. The CSI targeting routine iterates on the magnitude of a horizontal Δv at the CSI maneuver time. After each trial Δv, the active vehicle state is advanced the specified number of half periods to determine the time at which the CDH maneuver is to be performed. The required CDH maneuver is performed and the active state is then advanced to the specified TPI time. The elevation angle existing at TPI is compared with the required elevation angle and a change to the CSI Δv is computed by means of a Newton-Raphson iteration technique. The iteration process continues until the specified elevation angle is obtained at the specified TPI time. The output of the CSI targeting routine is the required horizontal Δv at CSI time, and the computed time at which CDH will occur.

There are two options available to the user with respect to computing the time of CDH in the CSI targeting routine. In both cases the conic period of the orbit is computed after impulsively applying the horizontal CSI Δv. For option one the time of CDH is then computed as the time of CSI plus the specified number of conic half periods. Thus if CSI is not at an apside of the orbit, CDH will also occur at a point other than at an apside of the orbit.

The second option also allows the user to initiate CSI at an arbitrary point on the orbit; however, CDH is constrained to occur at one of the apsides of the orbit resulting from the CSI maneuver. In this mode, the delta time from the CSI state to the nearest perifocus of the orbit is computed. This delta time is positive or negative respectively if the CSI state is approaching or has passed the nearest perifocus. The time of CDH is then computed as the time of CSI plus the specified number of half periods plus the delta time to nearest perifocus. This mode of operation is actually the original version of the on-board CSI/CDH targeting routine. If CSI should occur near apofocus of the orbit, then this mode of targeting can become unstable and result in no solution.

The CDH routine is called after performing the CSI maneuver with the time at which the CDH maneuver is to occur as an input parameter. The time of CDH was computed and output by the CSI targeting routine; however, it is possible for the crew to specify a CDH time which is different. In any case, the CDH routine calculates the Δv required for the active vehicle to go into a coelliptic orbit with respect to the target orbit at the specified CDH time.

A coelliptic orbit is one in which the differential altitude (ΔH) between the active and passive orbit, as measured along a radial line from the focus, is constant throughout the orbit. Taking ΔH to be positive for an active orbit altitude below the passive orbit altitude, and negative for a higher active orbit altitude, the following conditions must be satisfied at the conclusion of the CDH maneuver:

1. The semi-major axis of the active vehicle must be equal to the semi-major axis of the passive orbit minus the coelliptic altitude difference ΔH.

2. The product of the semi-major axis and eccentricity of the active vehicle orbit must equal the same product for the target orbit.

3. The line of apsides of the two orbits must be aligned.

Reference (4) gives an excellent derivation of the required conditions for obtaining a coelliptic orbit, and Reference (1) gives the equations as implemented in the on-board computer. In general, the CDH maneuver can have both a vertical and a horizontal component of Δv.

After achieving a coelliptic orbit, the TPI targeting routine is called to determine the Δv required for intercept. There are two end conditions which specify the TPI position: first, the desired line-of-sight elevation angle from the active to the passive vehicle, and second, the time of TPI. Both of these end conditions were satisfied in the CSI targeting. Due to possible execution or navigation errors in the CSI and CDH maneuvers, it may be impossible to simultaneously satisfy both end conditions at TPI. Thus, either the time of TPI, or the desired TPI elevation angle is input to the TPI targeting routine as the criterion for executing TPI. The other parameter is determined and output along with the required Δv at TPI for intercept.

One important point is that the on-board CSI/CDH targeting routines, and the UNIVAC 1108 version of these routines use conic routines other than precision integration routines. Thus the targeting results can be incorrect when applied to state vectors which are integrated with a full gravity model. This aspect of the problem is currently being investigated by the authors and will be reported in a later memorandum. In order to be consistent in this memorandum, all simulation runs involving use of the targeting routines have been performed using a central body gravitational model without oblateness and other perturbing effects.

III. Problem Description

A complete coelliptic rendezvous study begins with the specification of the nominal conditions required at TPI, i.e., the specified line-of-sight elevation angle and desired value of ΔH (the coelliptic height difference). These parameters are selected so that the terminal phase of the rendezvous will be as insensitive as possible to trajectory dispersions at the TPI point (see Reference 3).

Based upon previous rendezvous experiences, MSC rendezvous analyses, and the former AAP baseline mission, the following nominal coelliptic rendezvous conditions were selected for study:

1. target orbit is a 220 nm circular orbit,
2. coelliptic orbit 10 nm below the target orbit,
3. line-of-sight elevation angle at TPI of twenty-eight degrees, and

4. desired time in the coelliptic orbit, ΔT (the time from CDH to TPI), between thirty and sixty minutes.

The thirty-minute minimum time in the coelliptic orbit allows adequate time for tracking between the CDH and TPI maneuvers, while the sixty-minute maximum time prevents velocity errors at CDH from producing significant errors at TPI.

During the AAP rendezvous, the CSM will probably be inserted into an 81 x 120 nm orbit and may require up to sixteen orbits for rendezvous. Initial phasing and height maneuvers will be targeted by the ground and implemented by the CSM. At the conclusion of these initial maneuvers, the CSM should be able to complete the rendezvous using its on-board CSI/CDH and TPI targeting routines. The purpose of this study is then to define the range in relative phase and relative altitude of the active vehicle with respect to the passive vehicle, at the CSI maneuver, and the range of time from CSI to TPI for which the on-board targeting routines are effective. This will then define what accuracy is expected of the ground-targeted maneuvers.

It will be assumed here that all phasing and height maneuvers and the CSI maneuver will be performed at an apsidal crossing, and that, all of these maneuvers will consist of horizontal velocity changes. Further, the CSI/CDH targeting routine will operate in mode one as discussed in the previous section so that the CDH maneuver will also occur at an apsidal point.

IV. CSI/CDH Rendezvous Profiles

Figure 1 illustrates the two ways in which the CSI/CDH targeting routines can be used based upon the problem description of the previous section. This figure shows the trajectory of the active vehicle in a local vertical curvilinear system, centered in the target vehicle. The target vehicle is assumed to be in a circular orbit, and the horizontal axis represents the actual orbit of the target vehicle. This coordinate system will be used for all figures which illustrate relative position.

In Figure 1a, a height maneuver (NH) is performed which causes an apogee in the orbit of the active vehicle to occur at a specified distance, ΔH, below the orbit of the target vehicle. The CSI maneuver is to occur at this apogee and the CDH maneuver is to occur an even number of orbital half periods later. The figure illustrates the case of two half periods. For this profile, the coelliptic height difference (ΔH) is fixed by the NH maneuver, and the CSI maneuver is used only as a phasing maneuver to satisfy the desired terminal conditions at TPI.

In Figure 1b, a maneuver which can either be a phasing maneuver (NC1) or a height maneuver (NH) is performed first. Again, the CSI maneuver occurs at the apogee of the new orbit. The CDH maneuver will occur an odd number of half periods after CSI (shown as one half period in the figure). For this profile, the ΔH of the coelliptic orbit is adjusted automatically by the CSI iterations so that the end conditions at TPI will be satisfied. The profiles of Figures 1a and 1b will be referred to as constrained ΔH, and unconstrained ΔH coelliptic rendezvous respectively in the remainder of this memorandum.

V. Constrained ΔH Coelliptic Rendezvous

Figure 2 shows simulation results for a CSI/CDH targeted rendezvous for a CSI relative position of 245 nm behind the target vehicle and a coelliptic ΔH height of ten nautical miles. The figure illustrates the effect upon the resulting trajectory when the ΔT from CSI to TPI is varied.

For trajectory A, the ΔT from CSI to TPI is such that the required catch-up rate of the active vehicle is equal to the coelliptic catch-up rate. Thus, the CSI maneuver results in the active vehicle achieving a coelliptic trajectory and the vehicle coasts to TPI without a CDH maneuver.

For trajectory B, the ΔT from CSI to TPI is such that the required catch-up rate from CSI to CDH is equal to the catch-up rate prior to CSI. Thus, there is no change in velocity required at CSI and the Δv at CDH is such as to raise the perigee altitude to the coelliptic height.

For trajectory C, ΔT is greater than that for trajectory A; thus, the active vehicle must decrease its CSI-CDH catch-up rate relative to the coelliptic catch-up rate. To do this, the CSI maneuver increases the semi-major axis of the active vehicle to greater than the semi-major axis of the coelliptic orbit. The CDH maneuver is then performed an orbit later at the proper altitude and the vehicle coasts to TPI in the coelliptic orbit. The extra Δv at CSI required to raise the active vehicle apogee above the coellipti orbit altitude and then lower the apogee to the coelliptic height at CDH is a penalty resulting from improper phasing maneuvers prior to CSI. These earlier phasing maneuvers should have placed the CSI point further from the target, or caused the arrival at the given CSI position to be at a later time.

Trajectory D is also a result of improper phasing at CSI. The CSI maneuver must increase the catch-up rate between CSI and CDH over the catch-up rate prior to CSI. Thus the CSI maneuver results in lowering the perigee of the orbit. The CSI maneuver and the Δv at CDH to raise the perigee to its original height is a Δv penalty. In order to avoid the Δv penalty, the CSI point should have been closer to the target vehicle, or the CSI maneuver should have occurred earlier in time.

Trajectory E represents a desirable profile. The active vehicle must have a catch-up rate in the CSI to CDH portion of the trajectory which is greater than the coelliptic catch-up rate, but which is less than the catch-up rate prior to CSI. The CSI maneuver then raises perigee so that the resulting semi-major axis is less than the coelliptic semi-major axis, but greater than the original semi-major axis. Finally, the CDH maneuver raised the perigee the remainder of the distance to the coelliptic altitude.

The perigee altitude prior to CSI in Figure 2 has been chosen arbitrarily to illustrate Δv penalty. The significant point is that as long as the CSI maneuver raises perigee to some altitude between the original perigee altitude and the coelliptic altitude, and CDH raises the perigee altitude the remainder of the distance to the coelliptic altitude, then the total Δv at CSI and CDH will be constant regardless of the resulting intermediate perigee altitude. If the CSI maneuver must either lower perigee below the perigee altitude prior to CSI or raise the original perigee above the coelliptic altitude then the total Δv for the CSI and CDH maneuvers will increase. This can also be seen by referring to Figure 3 which illustrates the Δv penalty for the conditions shown in Figure 2. The required Δv's of this figure are based on conic trajectories. The CSI point is 245 nm behind the target vehicle, and the perigee altitude prior to CSI is 12 nm below the coelliptic altitude. The ΔT's corresponding to the trajectories of Figure 2 are indicated on Figure 3.

Figure 2 shows the situation when the CSI distance behind the target is fixed and ΔT varied. A similar set of curves results when ΔT is fixed and the CSI distance is varied.

Both of these results are combined in Figure 4, a plot of the time from CSI to TPI vs the distance the active vehicle is behind the target vehicle at CSI for the 10 nm coelliptic orbit. Contour lines of constant time in coelliptic orbit following the CDH maneuver, and constant relative perigee altitude in the CSI/CDH orbit are shown. Given ΔT and CSI distance, Figure 4 can be used to determine

-- the perigee altitude of the CSI/CDH orbit relative to the coelliptic altitude (Δr), and

-- the time spent in coelliptic orbit (DTCOE).

In Figure 4 the portions of the graph to the left of the solid contour line for $\Delta R = 10$ represent conditions for which the semi-major axis of the transfer orbit between CSI and CDH is larger than for the coelliptic orbit (trajectory C of Figure 2) and thus there is a Δv penalty. The portion of the graph below the solid contour line for DTCOE = 0, represent conditions for which there is insufficient time allowed to transfer from CSI to TPI (negative time is allowed for transfer from CDH to TPI), thus the CSI/CDH targeting routine is unable to obtain a solution. Figure 4 illustrates the desirability of having the perigee altitude prior to CSI as low as possible. This allows for a larger region in which the CSI/CDH maneuvers will not incur Δv penalty, and still allows for a large phasing capability. Finally it is seen that there is a relatively narrow ΔT band for which the time in coelliptic orbit constraint of thirty to sixty minutes is satisfied. Generally, the maximum limit of sixty minutes can be raised but the lower limit of thirty minutes is rather rigid.

All of the data discussed above were for the case of two half periods between CSI and CDH. As the distance behind at CSI and the time between CSI and TPI both increase, it becomes necessary to allow for more half periods between CSI and CDH. A targeted trajectory for four half periods is shown in Figure 5. All of the comments about Δv penalty discussed with Figure 2 are applicable to Figure 5. A set of contour lines similar to Figure 3 for four apsidal crossings is shown in Figure 6.

An important point related to CSI/CDH phasing adjustment is illustrated in Figures 2 and 4 and this point deserves special mention. In each case illustrated in Figure 2, the time interval from CSI to CDH is just the orbital period of the transfer orbit. For the trajectories shown, all orbital periods are approximately equal - being a minimum of 5522 seconds for trajectory D and a maximum of 5547 seconds for trajectory C. Further, even if the perigee of the CSI to CDH transfer orbit varies as much as 50 nm from the coelliptic altitude, the transfer time from CSI to CDH is within 5510±28 seconds. Thus, a good approximation of the time spent in coelliptic orbit is given by subtracting the approximate CSI to CDH time from the total CSI to TPI time. The approximate CSI to CDH time is given by multiplying 5510 by the number of full orbits from CSI to CDH.

Based upon the above approximation, and Figure 4, if the active vehicle is to preserve the time spent in coelliptic orbit (30 to 60 minutes) under the two half period flight plan, CSI apogee must occur respectively 7310 and 9110 seconds prior to TPI. Also, the CSI maneuver should be a minimum of 141 nm behind the target vehicle for 30 minutes in coelliptic orbit. For each additional second desired in coelliptic orbit, this minimum distance must be increased by 0.01695 nm in order to assure no Δv penalty. From the initial point, there is an additional allowable range of about 50 nm, of active vehicle distance behind the target vehicle, which can be made up with no Δv penalty for each 10 nm of difference between the coelliptic altitude and the pre-CSI perigee altitude.

If the distance behind at CSI is fixed, and the time from CSI to TPI is changed, Figure 4 shows that the perigee altitude from CSI to CDH will change by about 0.004 nm for each second change in the CSI to TPI time. This change results in a decrease of perigee altitude for a decrease in CSI to TPI time.

For the four half period flight plan (Figure 6), the CSI apogee must occur at 12820 and 14620 seconds prior to TPI for 30 and 60 minutes in coelliptic orbit respectively. The CSI point must be a minimum of about 237 nm behind the target vehicle for 30 minutes in coelliptic orbit, and this distance must increase by about 0.01615 nm for each additional second in coelliptic orbit in order to assure no Δv penalty. From the CSI points, there is then an additional range of about 100 nm of active vehicle distance behind the passive vehicle which can be made up with no Δv penalty for each 10 nm of difference between the coelliptic altitude and the pre-CSI perigee altitude.

The data of Figures 3 and 5 were prepared from simulation runs using the UNIVAC 1108 version of the CSI/CDH and TPI targeting routines. The data of Figures 4 and 7 were computed using conic equations rather than simulation runs; however, several cases were selected from Figures 4 and 7 and compared to simulation runs using the targeting routines. In all tests, the targeting routine solutions agreed.

VI. Unconstrained ΔH Coelliptic Rendezvous

The constrained ΔH profile of Figure 1a overcomes phase errors at CSI by changing the catch-up rate from CSI to CDH. In this way, the correct entry point into the coelliptic orbit is obtained and the correct TPI end conditions are achieved. The unconstrained ΔH profile of Figure 1b must overcome phase errors at CSI by either (a) adjusting the catch-up rate in coelliptic orbit by changing the coelliptic orbit altitude, or (b) by changing the time between CSI and TPI.

Figure 7 illustrates the effect upon the profile of Figure 1b when the time from CSI to TPI is varied for a fixed CSI point. The time from CSI to TPI was changed from 5000 seconds to 5900 seconds and this time difference of 900 seconds resulted in a change of the coelliptic altitude by 1.7 nm so as to slow the catch-up rate of the active vehicle. Comparison of Figures 2 and 7 illustrate the effect of errors in time of arrival at a fixed CSI point for the two profiles of Figure 1.

In Figure 7, the ratio of change of coelliptic altitude to change in time from CSI to TPI is 0.00189 nm/sec. However, if the CSI point had occurred further behind the target vehicle, with a corresponding increase in the CSI to TPI time, the sensitivity would be less. This is because there would have been a longer nominal time spent in coelliptic orbit, thus a smaller change in coelliptic catch-up rate over the longer time could produce the same change in CSI to TPI time.

Figure 8 shows the required CSI conditions for a one half period profile in order to insure a 10 nm ΔH coelliptic orbit. The scales are D_{CSI} - the distance behind at CSI, and ΔR - the distance of CSI below the target orbit. Contour lines for constant time in coelliptic orbit are shown. On a separate curve, the time from CSI to CDH ($T_{CSI/CDH}$) is shown as a function of ΔR. Given any two of the three variables at CSI ($T_{CSI/CDH}$, D_{CSI}, ΔR), Figure 8 can be used to determine the required value of the third variable so that the 10 nm ΔH characteristic is obtained.

As an example of the use of Figure 8, assume that CSI occurs 200 nm behind and 50 nm below the passive vehicle. From Figure 8, the time in coelliptic orbit is found to be 2400 seconds (from the family of D_{CSI} vs ΔR curves) and the time from CSI to CDH is 2747 seconds (from the $\Delta T_{CSI/CDH}$ vs ΔR curve). Thus the required time from CSI to TPI must be 5147 seconds in order to insure a ΔH of 10 nm.

The curves of Figure 8 are given by the following equations:

$$\Delta T_{CSI/CDH} = 2775.71 - 0.568 * \Delta R \tag{1}$$

$$D_{CSI} = 42.40 + 2.33\Delta R + 0.01695 * DTCOE, \tag{2}$$

and from these equations it is possible to draw the following general conclusions:

1. The time from CSI to CDH is approximately constant for a large range of ΔR. Thus a change in time from CSI to TPI results in an almost identical change in the time spent in coelliptic orbit.

2. In order to allow 1800 seconds in coelliptic orbit and maintain a ΔH of 10 nm, D_{CSI} and ΔR must satisfy the following relationship

$$D_{CSI} = 72.9 + 2.33\ \Delta R,$$

thus a minimum value for D_{CSI} is about 96 nm for $\Delta R = 10$ nm.

3. For each additional second desired in coelliptic orbit with $\Delta H = 10$ nm and a given value of ΔR, the distance behind at CSI must be increased by 0.01695 nm, and the time from CSI to TPI must also be increased by one second.

If off nominal conditions should occur at CSI, then the effect will be to change the coelliptic altitude, the entry point into the coelliptic orbit, and the time in coelliptic orbit. Table 1 shows the effect upon ΔH when a downrange error occurs at CSI (D_{CSI}) but the time to go ($\Delta T_{CSI/TPI}$)

is the required nominal value. The table is for ΔR over the range 10 to 100 nm and the nominal time $\Delta T_{CSI/TPI}$ and distance D_{CSI} are taken as that required for the specific value of ΔR and for ΔH = 10 nm. If the variation in D_{CSI} is such that the active vehicle is closer to the passive vehicle (variation is negative), then the coelliptic catch-up rate must be decreased and thus ΔH is decreased. The reverse is true if the variation in D_{CSI} is positive.

TABLE 1

Allowable variation in value of D_{CSI} for nominal value of $\Delta T_{CSI/TPI}$ and ΔH constrained to the limits [8, 12] nm, and [6, 14] nm

Nominal DTCOE (sec)	Allowable Variation in D_{CSI} (nm)	
	$\Delta H \varepsilon$ [8, 12] nm	$\Delta H \varepsilon$ [6, 14] nm
1800	± 14.56	± 29.11
3600	± 20.66	± 41.31
5400	± 26.76	± 53.51

It should be noted that the actual time in coelliptic orbit will be within about three seconds of the nominal time for the cases in Table 1. Note that symmetrical variations in D_{CSI} about its nominal value produces symmetrical variations in ΔH about its nominal value. Furthermore, the tolerance on variations in D_{CSI} varies linearly with the tolerance on ΔH.

If off nominal arrival time at CSI should occur, then again, the coelliptic altitude will be changed. Table 2 shows this effect, for the range of ΔR between 10 and 100 nm at CSI. For each case, the distance behind at CSI and time to go from CSI to TPI were first established in order to achieve the nominal ΔH of 10 nm and the indicated nominal times in coelliptic orbit. The table shows allowable maximum variation in the nominal time from CSI to TPI in order to hold ΔH with a value reasonably close to 10 nm. Since Table 2 does not exhibit the linearity nor the symmetry of Table 1, the data of Table 2 are plotted in Figure 9.

TABLE 2

Allowable variation in value of $\Delta T_{CSI/TPI}$ for nominal value of D_{CSI} and ΔH constrained to the Limits [8, 12] and [6, 14] nm

Nominal DTCOE	Allowable Variation in $\Delta T_{CSI/TPI}$	
	$\Delta H \varepsilon$ [8, 12] nm	$\Delta H \varepsilon$ [6, 14] nm
1800	[1073, -716]	[2864, -1227]
3600	[1524, -1016]	[4036, -1742]
5400	[1974, -1316]	[5264, -2256]

An increase in $\Delta T_{CSI/TPI}$ results in a required slower catch-up rate in coelliptic orbit and thus a decrease in ΔH. The reverse is true for a decrease in $\Delta T_{CSI/TPI}$. Also, any change in the time from CSI to TPI reflects directly into changing the time spent in coelliptic orbit. Thus the actual time spent in coelliptic orbit can be determined in Table 2 by adding the change in $\Delta T_{CSI/TPI}$ to the nominal value of DTCOE.

It is thus seen from Table 2 that extreme cases of time in coelliptic orbit can result. For example, if the nominal DTCOE is 1800 seconds and CSI occurs 1227 seconds late, the coelliptic altitude will be 14 nm with only about 573 seconds spent in coelliptic orbit.

The data of Figure 7 are from UNIVAC 1108 simulation runs using the CSI/CDH and TPI targeting routines. The data of Figure 8 and of Tables 1 and 2 are from conic equations rather than simulation runs. However, several of these cases were selected and test runs were made with the simulator. In all tests, the targeting routine solutions agreed.

VII. Conclusions

The constrained ΔH profile with two half periods between CSI and CDH has the following requirements in order to assure no Δv penalty:

1. CSI must occur a minimum of about 141 nm behind the target vehicle in order to allow 1800 seconds in coelliptic orbit.

2. The transfer time from CSI to CDH is essentially constant and can be approximated by 5510 seconds. Thus the minimum required time from CSI to TPI is about 7310 seconds in order to allow 1800 seconds in coelliptic orbit.

3. For each additional second desired in coelliptic orbit, the minimum distance behind at CSI must be increased by at least .01695 nm, and the total transfer time from CSI to TPI must be increased by one second.

4. The minimum distance behind at CSI can be increased by an additional range of 50 nm for each 10 nm of difference between the coelliptic altitude and the pre-CSI perigee altitude without changing either the total time from CSI to TPI or the time in coelliptic orbit.

The conclusions with respect to the unconstrained ΔH profile with one half period between CSI and CDH in order to achieve the nominal ΔH of 10 nm are:

1. CSI must occur a minimum of 96 nm behind the target vehicle in order to allow for 1800 seconds in coelliptic orbit.

2. The transfer time from CSI to CDH is essentially constant and can be approximated by 2755 seconds. Thus the minimum required time from CSI to TPI is about 4555 seconds in order to allow 1800 seconds in coelliptic orbit.

3. These two conclusions are for the case of CSI occurring 10 nm below the passive vehicle. For each additional nm decrease in the CSI altitude, the distance behind at CSI must increase by 2.33 nm and the time to TPI must decrease by about 0.6 seconds to insure ΔH = 10 nm and 1800 seconds in coelliptic orbit.

4. For each additional second desired in coelliptic orbit, the minimum distance behind at CSI must be increased by 0.01695 nm, and the transfer time from CSI to TPI must be increased by one second.

5. For a nominal time to go from CSI to TPI, variation in the nominal D_{CSI} will produce symmetrical variations in ΔH about its nominal, but will not change the nominal time spent in coelliptic orbit. This is true regardless of the distance below at CSI. Table 1 presented the data.

6. For a nominal distance behind at CSI, variations in the nominal $\Delta T_{CSI/TPI}$ produce unsymmetrical variations in ΔH about its nominal value, and directly changes the time spent in coelliptic orbit. This is true regardless of the distance below at CSI. Table 2 and Figure 9 presented this data.

The above data are for a target orbit of 220 nm with a 10 nm coelliptic altitude difference. The baseline for AAP is now for a target orbit of 235 nm; however, tests indicate that the above results are within three percent of the results for a 235 nm target orbit. The general conclusions are true for both 235 nm and 220 nm orbits.

C. O. Guffee Jr.

C. O. Guffee

R. C. Purkey

$1025-\frac{COG}{RCP}-1i$ R. C. Purkey

Attachments

References

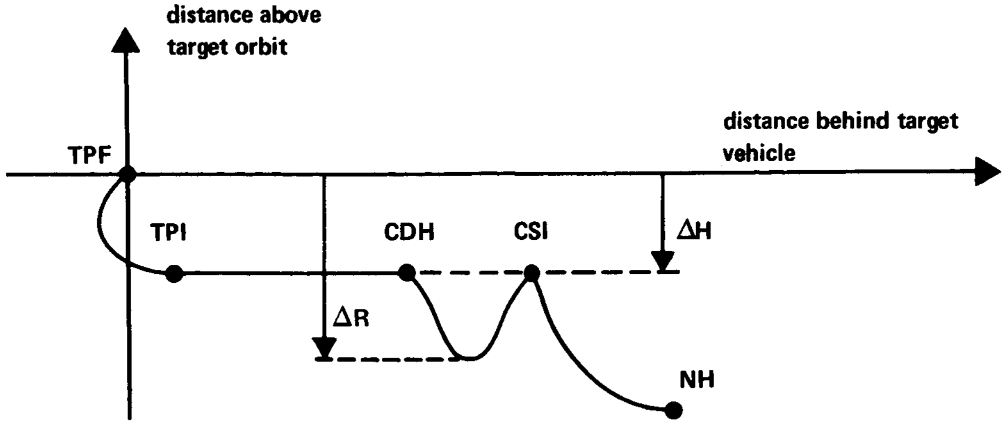
(1) Guidance System Operations Plan for Manned LM Earth Orbital and Lunar Missions Using Program Luminary (GSOP), Section 5 Guidance Equations, Revision 1, R-567, November 1968.

(2) J. A. Alexander, General Explanations of NCC/NSR and CSI/CDH Coelliptic Sequence Rendezvous Techniques, MSC Internal Note No. 68-FM-80, March 28, 1968.

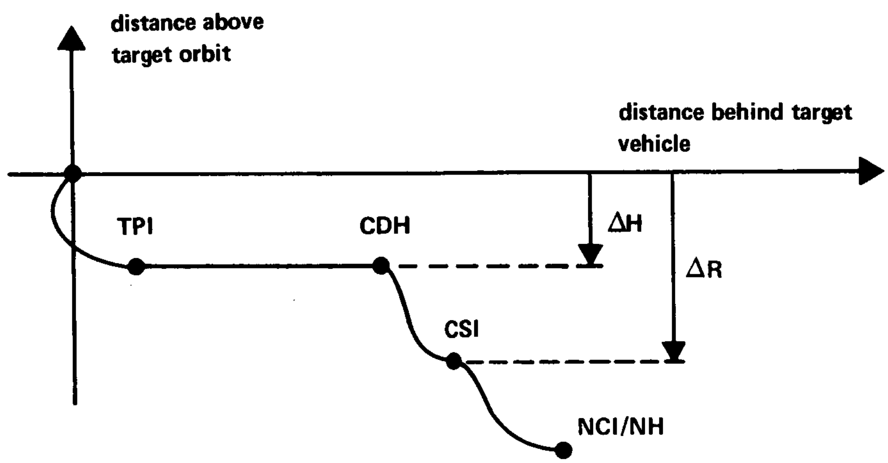
(3) Richard P. Parten and John P. Mayer, "Development of the Gemini Operational Rendezvous Plan", Journal of Spacecraft Volume 5, No. 9, September 1963, pp 1023, 1028.

(4) LM/AGS Flight Equations Narrative Description, 05952-6076-T000, prepared by TRW for MSC under Contract No. NAS9-4810, 25 January 1967.

(5) AAP Baseline Reference Mission, NASA MSC-KM-D-68-4A, 27 October 1967.



(a) CSI/CDH coelliptic rendezvous profile with constrained
ΔH altitude difference in coelliptic orbit.



(b) CSI/CDH coelliptic profile with unconstrained altitude
difference in coelliptic orbit.

FIGURE 1. POSSIBLE RENDEZVOUS PROFILE FOR CSI/CDH TARGETED RENDEZVOUS

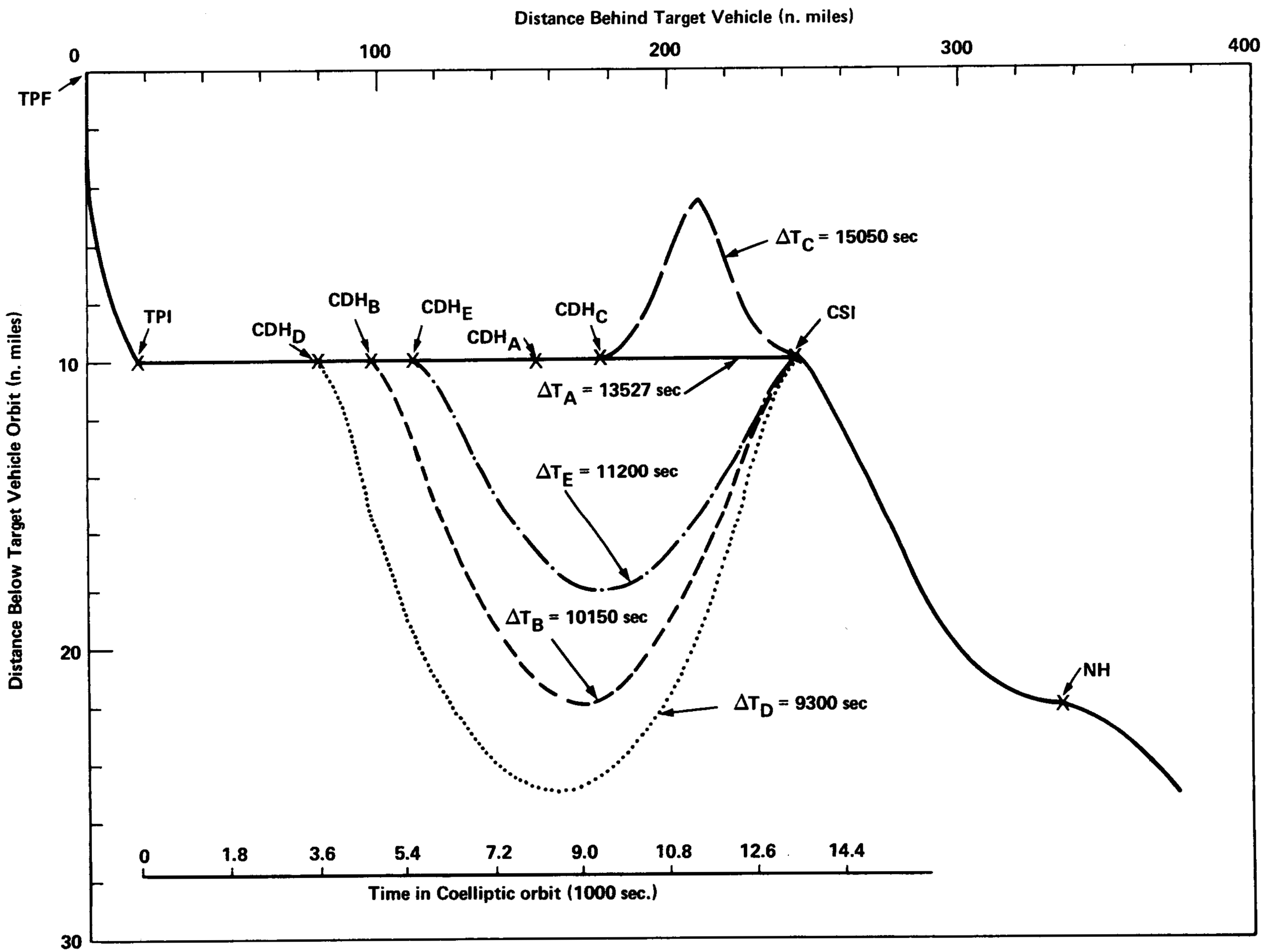


FIGURE 2. TYPICAL CSI/CDH TARGETED RENDEZVOUS PROFILES AS A FUNCTION OF Δ T FROM CSI TO TPI

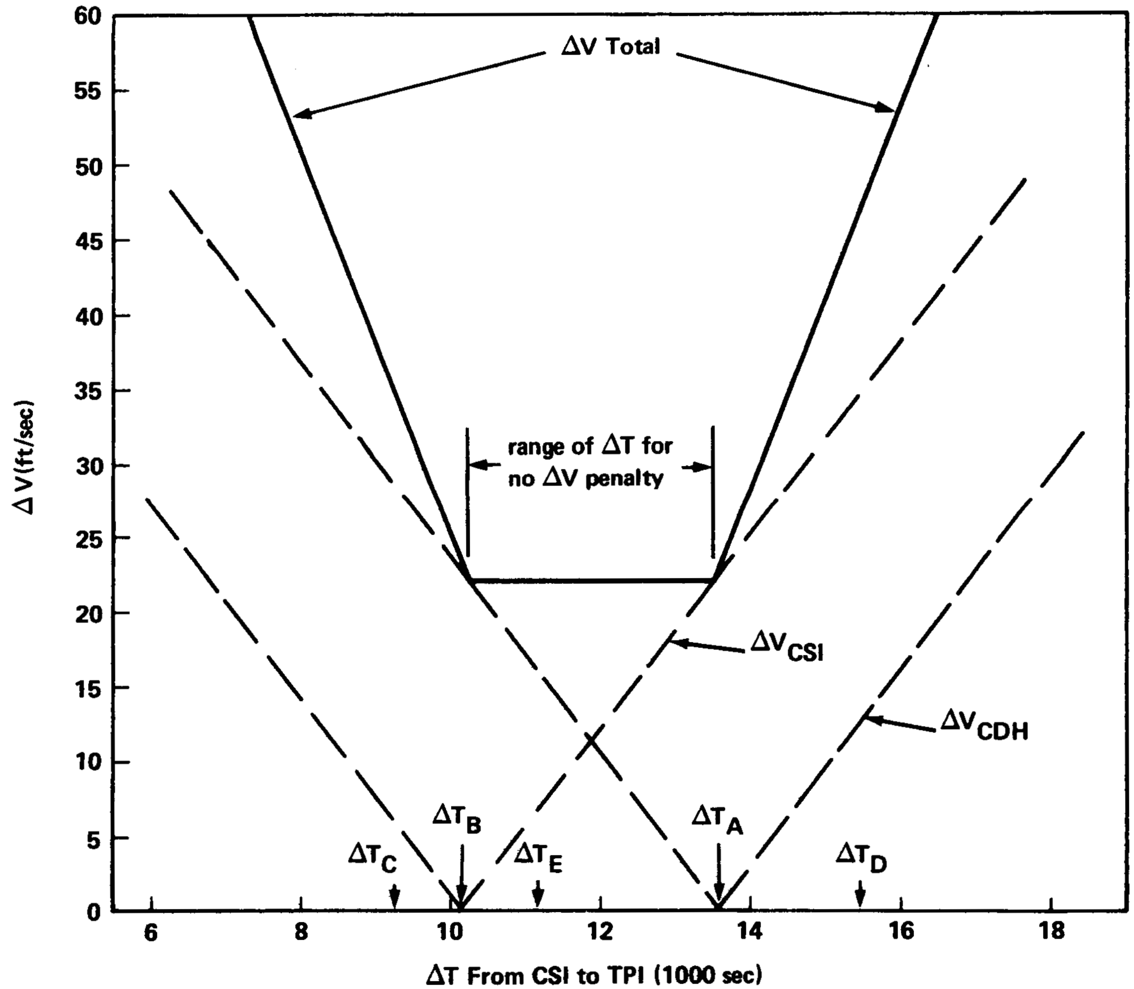


FIGURE 3. Δ V REQUIREMENTS FOR RENDEZVOUS PROFILES OF FIGURE 2

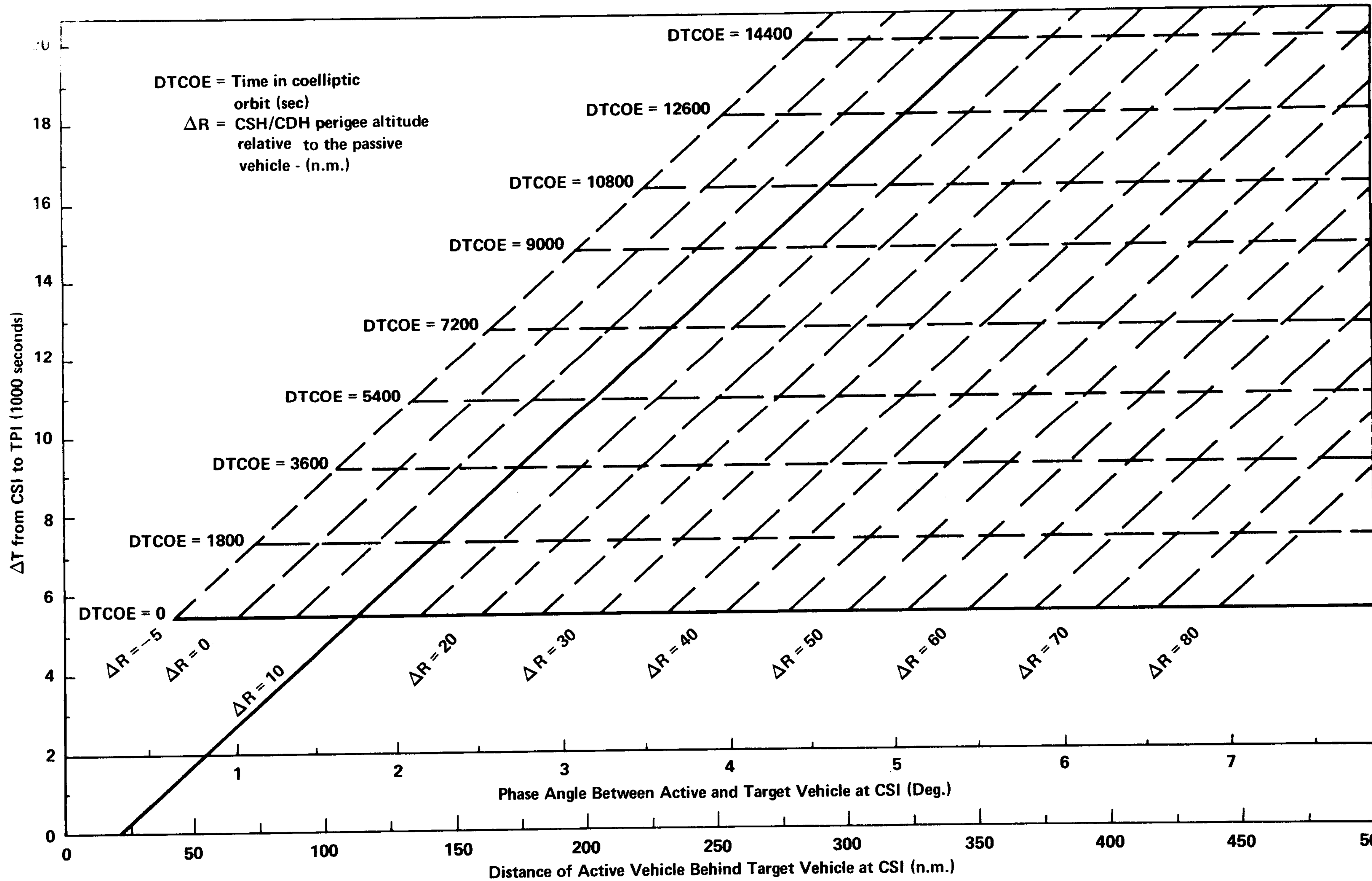


FIGURE 4. CONTOUR LINES OF TIME IN COELLIPTIC ORBIT AND CSI/CDH RELATIVE PERIGEE ALTITUDE FOR TWO APSIDAL CROSSINGS

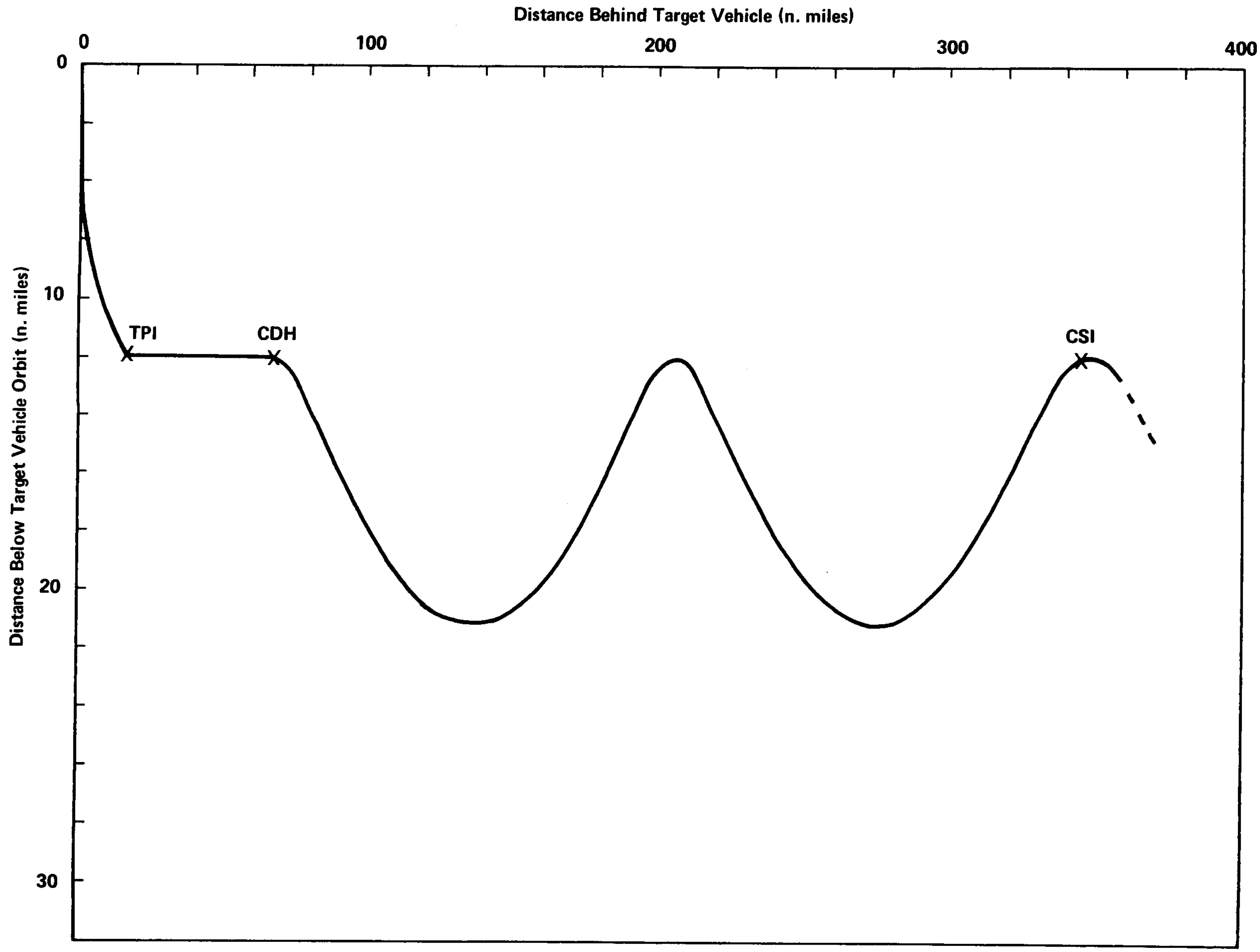


FIGURE 5. A TYPICAL CSI/CDH TARGETED RENDEZVOUS WITH FOUR APSIDAL CROSSINGS BETWEEN CSI AND CDH.

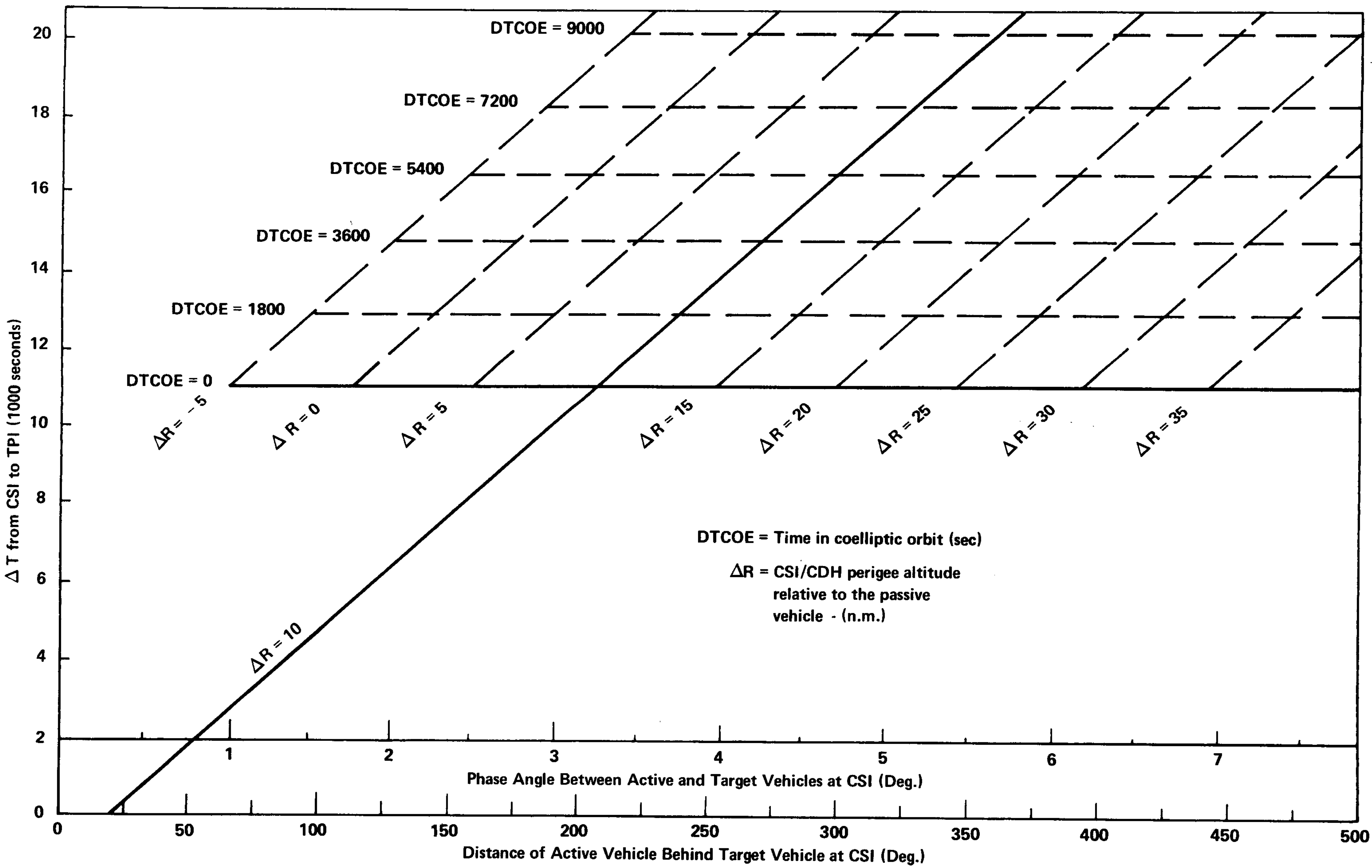


FIGURE 6. CONTOUR LINES OF TIME IN COELLIPTIC ORBIT AND CSD/CDH RELATIVE PERIGEE ALTITUDE FOR FOUR APSIDAL CROSSINGS

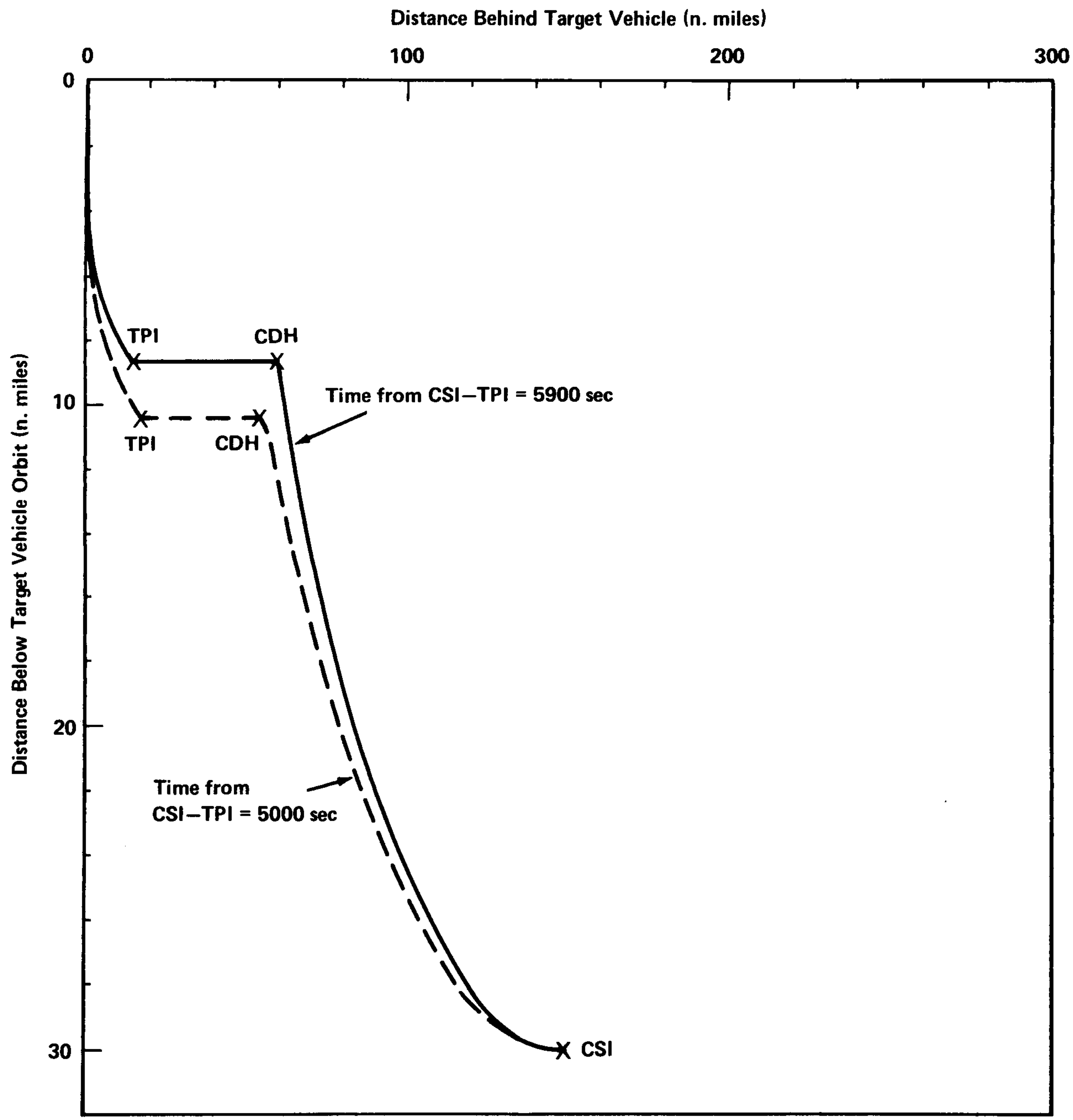


FIGURE 7. TYPICAL CSI/CDH TARGETED RENDEZVOUS FOR THE CASE OF ONE APSIDAL CROSSING FROM CSI TO CDH

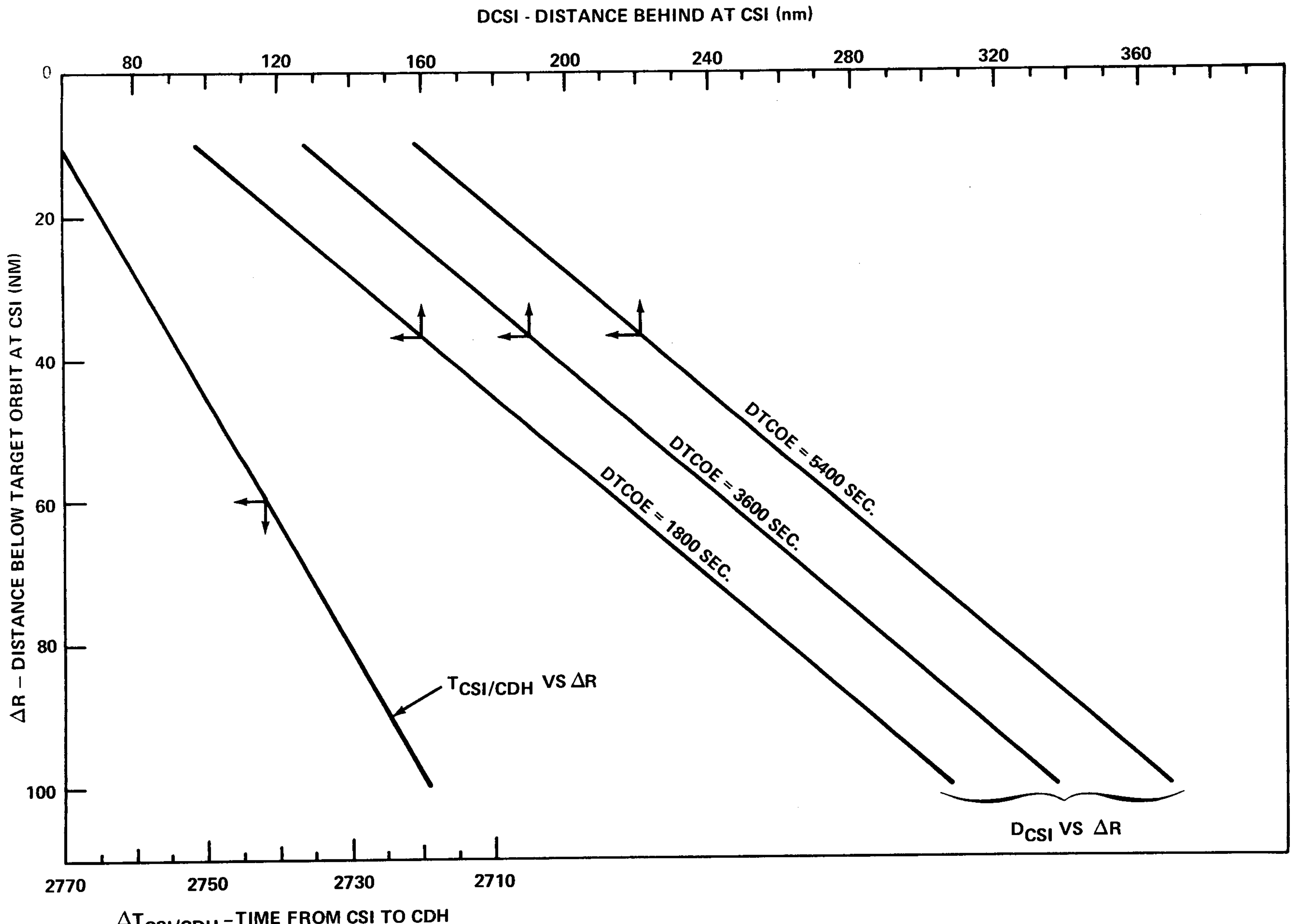


FIGURE 8 - BOUNDS ON CSI CONDITIONS FOR 10NM ΔH AND ONE HALF PERIOD FROM CSI TO CDH

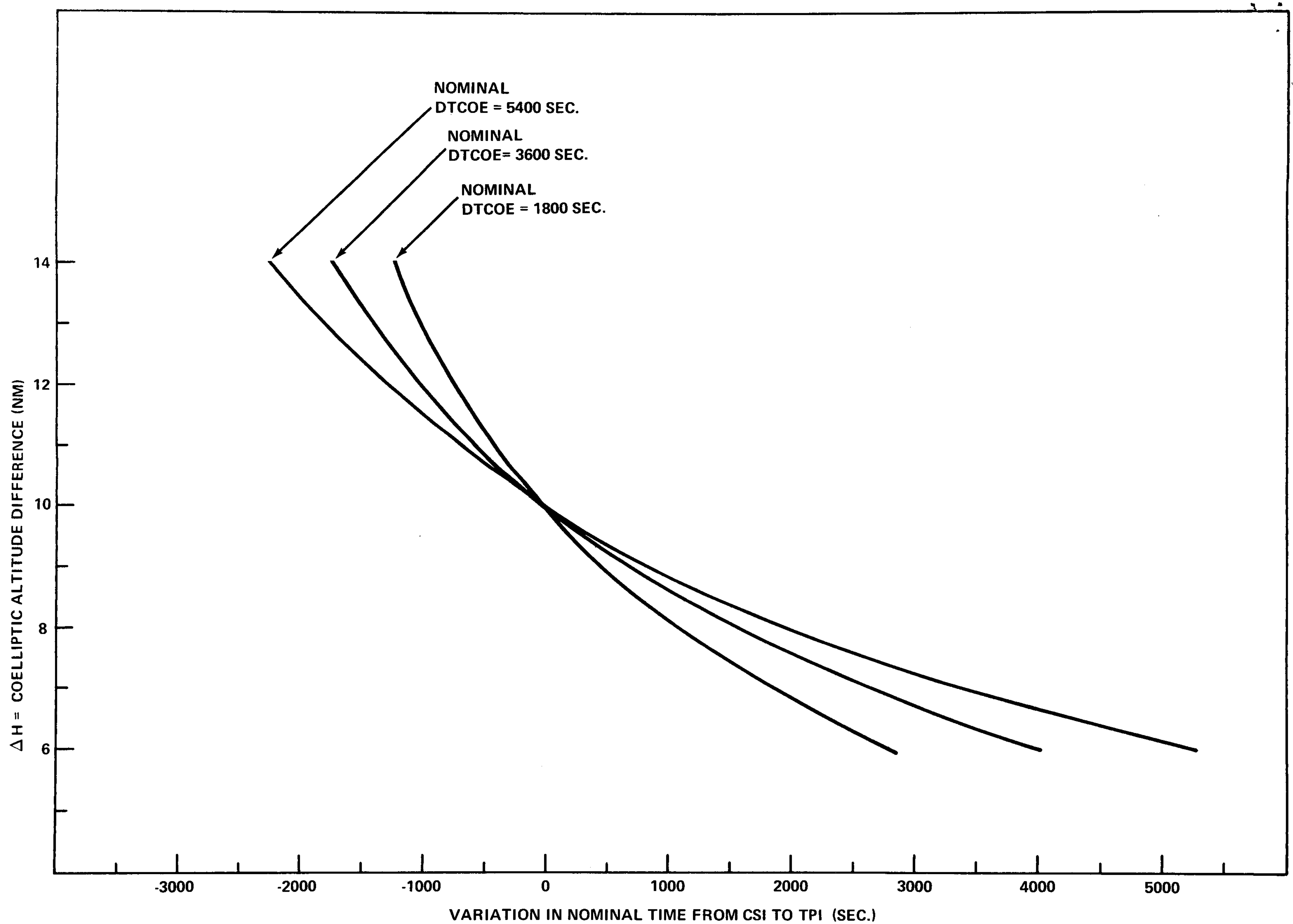


FIGURE 9 - VARIATION IN COELLIPTIC ALTITUDE DIFFERENCE VS. VARIATION IN NOMINAL TIME FROM CSI TO TPI FOR ONE HALF PERIOD PHASING MANEUVER.

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R. C. Purkey

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